Outsourcing timing, contract selection, and negotiation

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This paper examines outsourcing contracts subject to irreversible outsourcing investment and cost uncertainty. We consider three common outsourcing contracts (fixed-price, cost-plus, and gain-sharing) and address issues of when to outsource and which contracts to select. Motivated by both practical importance and lack of academic research in this area, this paper compares and analyzes these contracts’ utilities and outsourcing options’ values and assists managers in timing of outsourcing, and selection and negotiation of outsourcing contracts.

**Keywords**: Outsourcing; Buyer-supplier negotiations; Contract selection; Contract timing

1. INTRODUCTION

Outsourcing managers are facing practical issues relating to when to outsource, what outsourcing contract to select, and how to negotiate the outsourcing contract, subject to the irreversible outsourcing investment and uncertain future production and service cost. Motivated by both practical importance and lack of academic research in this area, this paper analytically compares three common outsourcing contracts – fixed-price, cost-plus, and gain-sharing. By analyzing these contracts’ utilities and outsourcing option values, we show that with complete information, none of the three considered here is necessarily superior to others under effective negotiation. In reality, however, the client normally does not have access to all the required information for effective negotiation. Based on our theoretical analysis established from limited information, we provide a practical guide for the client’s outsourcing timing, contract selection and negotiation.

While outsourcing may enhance a client firm’s values in several ways, such as access to state-of-the-art technology, improved quality, productivity and flexibility and focus on core competency, current research finds that cost reduction ranks the highest among other outsourcing
benefits (Bettis, Bradley & Hamel, 1992; Lei & Hitt, 1995; Li & Kouvelis, 1999; Quinn, 2000; Jiang, Frazier & Prater, 2006). A survey by Deloitte Consulting Group (2005) reveals that 70% of respondents mentioned cost savings as the primary reason for outsourcing. Other surveys from special areas of services confirm that cost reduction has the highest priority in client’s outsourcing decision making. For example, in the annual survey on business process outsourcing conducted by Capgemini and IDC (2006), executives cited reducing costs (41.2%), driving innovation (35.3%) and the ability to focus on core competencies (14.7%) as the main drivers when deciding to use outsourcing in a corporate strategy. According to Towers Perrin’s 2005 Human Resource (HR) Outsourcing Effectiveness Study (Zemke, 2005), HR managers focused on three key objectives in their outsourcing initiatives: reducing overall service delivery costs (37%), freeing HR to focus on more strategic issues (23%), and improving service quality (14%).

From the standpoint of considering outsourcing costs, cost-plus and fixed-price contracts have been the dominant contracting models of B2B services for some time (Lacity & Willcocks, 1998). However, both of these contracts have drawbacks in controlling costs effectively. The cost-plus contract requires detailed agreements that specify client requirements, vendor service levels and performance metrics, penalties for non-performance and pricing mechanisms. However, more often than not, it is very difficult to clearly define client’s baseline requirements. Client’s additional or undocumented services can raise vendor’s operating costs to a new level. Account managers in vendor firms are typically rewarded for maximizing profits, mainly for charging excess fee above clients’ baseline requirements. As a result, clients face increasing outsourcing expenses.

To limit the risk of unexpected charges, clients may negotiate fixed-price outsourcing contracts which fix all-inclusive fees for predetermined services. Such a contract can avoid vendor’s overcharges. However, it may depart significantly from the market price during the
contract duration. Lacity and Willcocks (1998) provide a case to describe such a situation. METAL, the client of a fixed-price and multiple-year IS outsourcing contract, agreed to pay $100 per processed form in 1990. But by 1993, the vendor was charging $50 per processed form to other customers in the open market. Because of the fixed-price contract, METAL had been unable to achieve the lower rate.

Thus, ongoing practices exhibit few options that can protect outsourcing firms from financial and technological risks. One of the more recent changes to outsourcing contracts is a trend toward “gain-sharing” contracts under which vendors share cost savings or cost overruns with clients. Should the vendor deliver the quality and service its client required at less than the targeted price, the client and the vendor will share the cost-savings. Should the vendor overrun the target price, the two parties share the overrun. Under a gain-sharing outsourcing contract, the client and the vendor may maximize the mutual benefit.

While the mechanism of the gain-sharing outsourcing contract seems intuitive and obvious, there is a critical impeding factor to implement such a contract, i.e., how to estimate the target price? For example, SHL Systemhouse Inc., an IS services vendor (Source: InformationWeek, June 26, 1996), initially estimated its client’s target outsourcing expenses at $5 million, but the client finally obtained the services for $4 million. The $1 million cost-savings was placed into a contingency pool. This caused a major issue because SHL wanted all the savings since they believed it be a bonus for their cost reduction efforts, but the client did not trust SHL’s original target price and was unwilling to forego the savings.

We develop an analytical model that considers the aforementioned outsourcing contracts in deciding outsourcing timing, contract selection and negotiation. Our model considers a client and a vendor with irreversible outsourcing investment cost and uncertain future cost. The client
decides on the timing to outsource, the contract format to utilize, and negotiates the contract with
the vendor.

Existant literature has paid little attention to clients outsourcing contract selection, negotiation and timing. This research aims to fill this gap through an analytical study of outsourcing contract selection under cost uncertainty. The paper is organized as follows. The next section provides a brief literature review on outsourcing decision-making and contract valuation. The following section presents the basic setting for this research. We then compare the three contracts’ utilities and prices, which is followed by analyzing the outsourcing timing through a real options approach. Managerial insights to clients on outsourcing contract selection and negotiation are provided next and the paper ends with conclusions and extensions to this research.

2. LITERATURE REVIEW
There are few qualitative studies relating to outsourcing contracts selection in existant literature.

For example, Auguste, Hao, Singer and Wiegand (2002) argue that the vendor lacks any incentive to reduce costs with a costs-plus contract; with a fixed-price contract, the vendor gains rewards from process innovation by keeping costs under control. As a result, the fixed-price contract is a better choice for the client. In contrast, Kern, Willcocks and van Heck (2002) point out that a fixed-price contract will create inflexibilities, possibly disadvantageous to both parties. They conclude that flexible contracts, e.g., cost-plus and gain-sharing, can help clients and vendors avoid the notorious “winner’s curse” in outsourcing contract bids. In a field study, Lacity and Willcocks (1998) found that only when the client and vendor shared risks and gains, i.e., signed a gain-sharing contract rather than a fixed-price contract, the client could benefit from vendor’s cost reduction.

These conflicting conclusions create a need for analytical comparison, similar to Cachon and Lariviere’s (2005) thorough investigation of several types of contracts in supply chain
management, to explore the insights into the three common outsourcing contracts. While the contracts in Cachon and Lariviere’s (2005) study are closely related and have the same contract titles (such as fixed price and revenue sharing) of the outsourcing contracts we are interested in, the fundamental difference between the two studies is that Cachon and Lariviere focused on purchasing, where our focus is on outsourcing.

Gilley and Rasheed (2000) point out that defining outsourcing simply in terms of procurement activities does not capture the true strategic nature of the issue. Outsourcing is not simply a purchasing decision, because all firms purchase elements of their operations. Outsourcing may arise through the substitution of external purchases for internal activities. In this way, it can be viewed as a discontinuation of internal production or service and an initiation of procurement from outside suppliers.

To the extent that this type of outsourcing reduces a firm’s involvement in successive stages of production, substitution based outsourcing may be viewed as vertical disintegration. Sometimes, however, a firm purchases goods or services from outside organizations when those goods or services have never been made in-house in the past. In other words, organizations have no choice but to acquire particular goods or services from external sources. Gilley and Rasheed define this situation as abstention, and deny that it is a qualified outsourcing.

Based on the transaction cost theory, when a firm has already integrated its operational functions, the decision to outsource some functions to the market should be made if it is necessary to create or protect the firm’s value (Stuckey & White, 1993). Once a previous in-house operation is contracted out, the client has, to some extent, lost the internal technical expertise and possibly

* Laffont and Tirole (1993) constructed Stackelberg game to consider a client as well as a vendor as utility maximizers as to efficiency and effort in the incentive contract framework. However, the discontinuation of internal production was not considered in their analysis.
dismantles and sells off in-house facilities (Weidenbaum, 2005). This carries potential risks if the vendor cannot or is unable to provide the outsourcing service economically (Harland, Knight, Lamming & Walker, 2005). Decisions to dismantle internal activities damage the economics of in-house operations in ways that make them costly to rebuild (Aron, Clemons & Reddi, 2005). For example, to recreate an IS facility, the outsourcing firm will generally incur capital costs of all new assets, whereas before it could have used depreciated assets at relatively low costs in terms of further depreciation (decline in market value) and opportunity cost (foregone interest on market value tied up in assets). For this reason, a logical principle in financial economics is that no decision should be undertaken unless its net benefits at least compensate for the loss of “option to wait” (Dixit, 1989; McGrath, 1997; Huchzermeier & Loch, 2001).

Dixit and Pindyck (1994) discuss optimal investment timing in the framework of irreversibility and uncertainty, and point out the parallels between investment opportunities and call options and, as a consequence, the existence of opportunity costs which fundamentally influence the decision-making behavior. In most cases, the outsourcing time is not exogenously fixed. The outsourcing firm has an option to exercise its outsourcing opportunity immediately or hold this opportunity for sometime. An outsourcing firm which has outsourced its previous in-house operations can no longe hedge since the option to wait is foregone. Conventionally, investment is seen as acceptable when NPV (net present value) exceeds zero. The variation on this rule dictated by real options theory is that any action that reduces managerial flexibility in ways which may prove very costly, is acceptable only if its NPV exceeds the financial value of those lost options. Current literature has been increasingly applying option models to capture total values in irreversible situations and argues that the real options approach is much better than the traditional NPV approach in decision-making (e.g., see Copeland & Keeney, 1998; Luehrman, 1998; Johnstone, 2002; Kamrad & Siddique, 2004). In the literature on outsourcing, the application of
real options theory has been growing as well. For example, Johnstone (2002), Nembhard, Shi and Aktan (2003), and Alvarez and Stenbacka (2006) have considered outsourcing as an option that clients have to analyze their outsourcing decisions. In contrast, our work studies various forms of outsourcing contracts, negotiation, and timing.

There is a growing literature in supply chain management that examines outsourcing contracts. Van Mieghem (1999) examined subcontracting and outsourcing conditions for price-only contracts and incomplete contracts. He defines outsourcing as a special case of subcontracting with no in-house production capability. Plambeck and Taylor (2005) analyzed the selling of production facilities by original equipment manufactures to contract manufacturers and its impact on investment in capacity and innovation. Cachon and Lariviere (2005) examined revenue-sharing contracts within a general supply chain setting. Ren and Zhou (2008) studied contracting issues specific to service outsourcing, in particular, a call center. See Cachon (2003) for an extensive discussion and survey of supply chain contracts, and Nagarajan and Sosic (2008) for a review of cooperative game theory and bargaining in supply chain management. In this paper, we study contracts when firms consider outsourcing to replace in-house production. Our work also examines both irreversible outsourcing investment under uncertainty and supply chain contract negotiation. This work adds to the body of knowledge in outsourcing by effectively considering issues relating to timing, contract selection and negotiation.

3. THE SETTING
We consider a three-stage timeline. In the first stage \((t < t^-_0)\), a client runs an in-house service. The second stage is characterized by three decision points \(t^-_0, t^+_0, \text{ and } t^-_0\). At \(t^-_0\), the client selects one of the three outsourcing contracts (fixed-price, cost-plus, and gain-sharing); at \(t^+_0\), the client negotiates this contract with a vendor; at \(t^-_0\), the client decides whether to give up the option to wait,
i.e., outsource the previous in-house operation to the vendor now, given the results of contract selection and negotiation. It is worth to note that the three decision points actually overlap with each other in reality. By breaking up these simultaneous decisions into parts, we significantly gain in expositonal efficiency. In the third stage ($t > t_0^+$), the vendor provides the agreed service to the client and charges a transfer price $P_t$. A graphical illustration of the sequence of events is provided in Figure 1.

![Figure 1. Sequence of events](image)

We now describe the features of outsourcing by the following assumptions:

(1) Operating costs $W_t$. Operating costs for the client and the vendor are $W_C$ and $W_V$, respectively. $W_v = W_t$, $W_C = W_t / \lambda$, for $\lambda \in (0,1]$. We first consider a simple setting by assuming that $W_t$ evolves over time as $dW_t = \mu W_t dt$, where $\mu$ implies the shift rate of expected future change. In section 5, we will extend to a general setting that $W_t$ evolves over time as a general Brownian motion (GBM).

(2) When the client terminates its in-house operation, the client has to incur a liquidation cost $K_C$.

(3) Client’s internal transaction price of the previous in-house operation is represented by $P_C$.

(4) Client’s utility with outsourcing: $U_C(t) = E\left[\int_0^t (P_C - P_t)e^{-rt} d\tau\right]$. $r$ is the capital cost and is positively related to the real interest rate and the industry specific risk rate. Such an infinite time horizon with exponential discounting does not change the essence of our results, but gives us mathematical tractability. We also assume $\mu < r$ for this setting; otherwise the integral of $dW_t$ would become indefinitely large.
Vendor’s utility with outsourcing: 
\[ U_V(t) = E\left[ \int_t^{\infty} (P_t - W_r) e^{-\tau} d\tau \right] . \]

Vendor’s price under the fixed-price outsourcing contract: 
\[ P_t = P_{FP}, \text{ a constant determined at } t_0, \text{ right before the time the contract is signed.} \]

Vendor’s price under the cost-plus outsourcing contract: 
\[ P_t = P_{CP} = (1 + \alpha_{CP})W_r, \text{ with a negotiated profit margin } \alpha_{CP}. \]

Vendor’s real price under the gain-sharing outsourcing contract is not the nominal price \( P_{TG} \) in the contract but the actual price \( P_{GS} \) on the delivery date which reflects the gain-sharing between the vendor and the client. As a result, the real price is 
\[ P_t = P_{GS} = P_M + (P_{TG} - P_M)s, \]
given a negotiated target price \( P_{TG} \), the market price \( P_M \), and the vendor’s share ratio \( s \in [0,1] \) of the gain (consequently, the client’s share ratio is \( 1-s \)). In the outsourcing market, 
\[ P_M = (1 + \alpha)W_r. \]
As a result, \( P_{GS} = P_{TG} s + (1-s)(1+\alpha)W_r. \) The outsourcing market’s profit margin \( \alpha \) is assumed to be exogenously given.

Vendor’s negotiation power is \( \gamma \in [0,1] \). Consequently, the client’s negotiation power is \( 1 - \gamma \).

4. CHOOSE AND NEGOTIATE OUTSOURCING CONTRACTS

If the client and the vendor reach an agreement on outsourcing, then the client outsources the previous in-house operation and incurs a liquidation cost \( K_C \), and the vendor serves the client with an operating cost \( W_V \) and charges \( P_t \). If they fail to reach agreements on outsourcing, each firm earns a default disagreement payoff. The vendor’s default payoff is assumed to be zero, while the client keeps that operation in-house with a default in-house operating payoff. We formulate the negotiation in the form of General Nash Bargaining (GNB) problem (Nagarajan & Sosic, 2008; Gurnani & Shi, 2006):

\[
\text{arg max } B(.) = [U_V - D_V][U_C - D_C]^{1-\gamma}, \tag{1}
\]

where \( U_i \) and \( D_i \) are the agreement and disagreement values, respectively, for \( i = V,C. \)

\[
D_V = 0, \tag{1a}
\]

\[
D_C = E\left[ \int_t^{\infty} (P_C - W_r / \lambda) e^{-\tau} d\tau \right] = \frac{P_C}{r} - \frac{W_r}{(r-\mu)\lambda}. \tag{1b}
\]

For the three outsourcing contracts, the client’s and vendor’s utilities can be described as following:
The fixed-price contract:

\[ U_{VP}^{FP}(t) = E \int_{t}^{\infty} (P_{FP} - W_\tau)e^{-r(\tau-t)}d\tau = \frac{P_{FP}}{r} - \frac{W_\tau}{r - \mu}, \]  

(2)

\[ U_{CP}^{FP}(t) = E \int_{t}^{\infty} (P_{C} - P_{FP})e^{-r(\tau-t)}d\tau = K_c = \frac{P_{C} - P_{FP}}{r} - K_c. \]  

(3)

The cost-plus contract:

\[ U_{VP}^{CP}(t) = E \int_{t}^{\infty} (P_{CP} - W_\tau)e^{-r(\tau-t)}d\tau = \alpha_{CP} \frac{W_\tau}{r - \mu}, \]  

(4)

\[ U_{CP}^{CP}(t) = E \int_{t}^{\infty} (P_{C} - P_{CP})e^{-r(\tau-t)}d\tau - K_c = \frac{P_{C}}{r} - K_c - (1 + \alpha_{CP}) \frac{W_\tau}{r - \mu} \]  

(5)

The gain-sharing contract:

\[ U_{VP}^{GS}(t) = E \int_{t}^{\infty} (P_{GS}^* - W_\tau)e^{-r(\tau-t)}d\tau = E \int_{t}^{\infty} (P_{GS} - (1-s)(1+\alpha)W_\tau - W_\tau)e^{-r(\tau-t)}d\tau \]

\[ = \frac{P_{TG}^*}{r} + [(1-s)(1+\alpha) - 1] \frac{W_\tau}{r - \mu}, \]  

(6)

\[ U_{CP}^{GS}(t) = E \int_{t}^{\infty} (P_{C} - P_{GS}^*)e^{-r(\tau-t)}d\tau - K_c = E \int_{t}^{\infty} (P_{C} - P_{TG}^* - (1-s)(1+\alpha)W_\tau)e^{-r(\tau-t)}d\tau - K_c \]

\[ = \frac{P_{C} - P_{TG}^*}{r} - K_c - (1-s)(1+\alpha)W_\tau. \]  

(7)

Because the outsourcing benefits should surpass the outsourcing costs, we let \( K_c \leq \frac{W_\tau}{(r-\mu)} (\frac{1}{\lambda} - 1) \).

From Eq. (1) we can calculate the fixed-price contract’s optimal negotiated price, the cost-plus contract’s optimal negotiated profit margin, and the gain-sharing contract’s optimal negotiated target price as follows (See Appendix A):

\[ P_{FP}^* = (1 + \gamma(\frac{1}{\lambda} - 1)) \frac{r}{r - \mu} W_\tau - \gamma K_c r, \]  

(8)
\[\alpha_{CP}^* = \gamma(-K_C \frac{r-\mu}{W_b} + \frac{1}{\lambda} - 1), \quad (9)\]

\[P_{CG}^* = \frac{r}{s} \left[ \left(1 + \gamma \frac{1}{\lambda} - 1 \right) s - \gamma(\frac{1}{\lambda} - 1) \right] \frac{W_b}{r-\mu} - K_C \gamma \right]. \quad (10)\]

From Eqs. (9 and 10), we get the cost-plus contract and the gain-sharing contract’s prices as follows:

\[P_{CP}^* = \left[(1 + \gamma \frac{1}{\lambda} - 1) \right] \frac{r}{r-\mu} W_b - \gamma K_C r = P_{FP}^* \frac{r-\mu}{r}, \quad (9a)\]

\[P_{GS}^* = (1-s)(1+\alpha) \frac{-\mu}{r-\mu} W_b + (1+\gamma \frac{1}{\lambda} - 1) \frac{r}{r-\mu} W_b - \gamma K_C r = P_{FP}^* - (1-s)(1+\alpha) \frac{\mu}{r-\mu} W_b. \quad (10a)\]

Considering Eqs. (8, 9, and 10), we simplify Eqs. (2) to (7) as follows:

\[U_v^{FP}(t_0) = U_v^{CP}(t_0) = U_v^{GS}(t_0) = \gamma \left[ \frac{1}{\lambda} - 1 \right] \frac{W_b}{r-\mu} - K_C \right]. \quad (11)\]

\[U_c^{FP}(t_0) = U_c^{CP}(t_0) = U_c^{GS}(t_0) = \left( \frac{P_C}{r} - \frac{1}{\lambda} \right) \frac{W_b}{r-\mu} + (1-\gamma) \left[ \frac{1}{\lambda} - 1 \right] \frac{W_b}{r-\mu} - K_C \right]. \quad (12)\]

These results reveal that the format of contract (fixed-price, cost-plus, or gain-sharing) will not affect the client’s expected utility given the condition that the client can effectively negotiate the optimal fixed price, profit margin, or target price. While under an effective negotiation the three outsourcing contracts bring the same utility to the client, asymmetric information in reality may obstruct the client to reach the optimal negotiated results. Thus, clients need some practical guidelines to direct their contract selection and negotiation.

Now we study the relations among the three optimal outsourcing contracts’ prices under different scenarios of cost and vendor competition, i.e., various \(\mu\) and \(\alpha\). From Eqs. (8, 9a, and 10a), we can compare \(P_{FP}, P_{CP},\) and \(P_{GS}\) directly and get Proposition 1 (See Appendix B):
Proposition 1:

\[ s + \gamma(-K_c \frac{r - \mu}{W_{h_0}} + \frac{1}{\lambda} - 1) \]

If \( \mu > 0 \), \( \alpha \leq \frac{1}{1 - s} \) \( P_{FP}^* > P_{GS}^* \geq P_{CP}^* \);

If \( \mu < 0 \), \( \alpha \leq \frac{1}{1 - s} \) \( P_{GS}^* \geq P_{CP}^* > P_{FP}^* \);

If \( \mu = 0 \), \( P_{FP}^* = P_{CP}^* = P_{GS}^* \);

If \( \mu > 0 \), \( \alpha > \frac{1}{1 - s} \) \( P_{FP}^* > P_{CP}^* > P_{GS}^* \);

If \( \mu < 0 \), \( \alpha > \frac{1}{1 - s} \) \( P_{GS}^* \geq P_{CP}^* > P_{FP}^* \);

Proposition 1 reveals how to select the best outsourcing contract when the client only has partial information on cost and competition. Based on Proposition 1, we get Theorem 1.

Theorem 1 (contract selection):

(i) The fixed-price contract

When the expected cost increases (\( \mu > 0 \)), the fixed price is most expensive among the three contracts; when the expected cost decreases (\( \mu < 0 \)), the fixed price is cheapest.

(ii) The cost-plus contract

When the expected cost increases and vendor competition is strong, the cost-plus price is cheapest; when the expected cost decreases and vendor competition is weak, the cost-plus price is most expensive.

(iii) The gain-sharing contract
When the expected cost increases and vendor competition is weak, the gain-sharing price is cheapest; when the expected cost decreases and vendor competition is strong, the gain-sharing price is most expensive.

According to utility functions Eqs. (2 to 7), we can get Theorem 2 as follows:

**Theorem 2 (vendor’s incentive):**

The fixed-price contract encourages the vendor to reduce operating costs, but the cost-plus contract does not; the gain-sharing contract encourages the vendor to reduce operating costs as long as the vendor grasps a large enough share ratio in this contract. The vendor’s cost reduction endeavors never damage the client’s utility.

Proof: From Eqs. (2, 4, and 6), \( \frac{\partial U_{FP}^V}{\partial W_i} = -\frac{1}{r-\mu} < 0 \), \( \frac{\partial U_{CP}^V}{\partial W_i} = \frac{\alpha_{CP}}{r-\mu} > 0 \), and

\[
\frac{\partial U_{GS}^V}{\partial W_i} = \frac{\alpha - s - s\alpha}{r-\mu} > 0 \quad \text{if and only if} \quad s < \frac{\alpha}{1+\alpha}.
\]

From Eqs. (3, 5, and 7), \( \frac{\partial U_{FP}^C}{\partial W_i} = 0 \),

\[
\frac{\partial U_{CP}^C}{\partial W_i} = -\frac{(1+\alpha_{CP})}{r-\mu} < 0, \quad \text{and} \quad \frac{\partial U_{GS}^C}{\partial W_i} = -\frac{(1-s)(1+\alpha)}{r-\mu} < 0.
\]

According to the client’s utilities under the three outsourcing contracts (Eqs. 3, 5, and 7), we obtain: \( \frac{\partial U_{FP}^C}{\partial P_{FP}} < 0; \frac{\partial U_{CP}^C}{\partial \alpha_{CP}} < 0; \) and \( \frac{\partial U_{GS}^C}{\partial P_{TG}} < 0 \), i.e., the client’s utility will increase if the client can negotiate a lower fixed price \( P_{FP} \), a lower vendor’s profit margin \( \alpha_{CP} \), or a lower target price \( P_{TG} \) for a given share ratio \( s \). To help clients understand their appropriate negotiation strategies on \( P_{FP}, \alpha_{CP}, \) and \( P_{TG} \), we investigate how \( P_{FP}, \alpha_{CP}, \) and \( P_{TG} \) change with potential influences: (i) from the vendor’s side \( (W_i \text{ and } \mu) \), (ii) from the client’s side \( (P_C \text{ and } K_C) \), (iii) from the interaction between vendor and client \( (\lambda \text{ and } \gamma) \), and (iv) from external environment \( (r \text{ and } \alpha) \).
Based on the optimal negotiated results Eqs. (8, 9, and 10), we obtain Proposition 2 as follows:

**Proposition 2:**

**For fixed-price contract:**

(i) \( \frac{\partial P_{FP}^*}{\partial W_t} > 0, \frac{\partial P_{FP}^*}{\partial \mu} > 0; \)

(ii) \( \frac{\partial P_{FP}^*}{\partial P_c} = 0, \frac{\partial P_{FP}^*}{\partial K_c} < 0; \)

(iii) \( \frac{\partial P_{FP}^*}{\partial \lambda} < 0, \frac{\partial P_{FP}^*}{\partial \gamma} \begin{cases} 
\geq 0, \text{if } K_c \leq \frac{W_t}{(r - \mu) \frac{1}{\lambda} - 1} \\
< 0, \text{if } K_c > \frac{W_t}{(r - \mu) \frac{1}{\lambda} - 1}
\end{cases}; \)

(iv) \( \frac{\partial P_{FP}^*}{\partial r} < 0, \frac{\partial P_{FP}^*}{\partial \alpha} = 0. \)

**For cost-plus contract:**

(i) \( \frac{\partial \alpha_{CP}^*}{\partial W_t} > 0, \frac{\partial \alpha_{CP}^*}{\partial \mu} > 0; \)

(ii) \( \frac{\partial \alpha_{CP}^*}{\partial P_c} = 0, \frac{\partial \alpha_{CP}^*}{\partial K_c} < 0; \)

(iii) \( \frac{\partial \alpha_{CP}^*}{\partial \lambda} < 0, \frac{\partial \alpha_{CP}^*}{\partial \gamma} < 0; \)

(iv) \( \frac{\partial \alpha_{CP}^*}{\partial r} < 0, \frac{\partial \alpha_{CP}^*}{\partial \alpha} = 0. \)

**For gain-sharing contract:**
Proposition 2 reveals that without enough information to negotiate optimal results, the client can use partial information to steer the negotiation of $P_{TP}, \alpha_{CP}$ or $P_{TG}$ towards the right direction. According to Propositions 2, we have Theorem 3:

**Theorem 3 (contract negotiation):**

(i) **Influence from vendor side**

*When the vendor’s operating cost is expected to increase, the client prefers to select the gain-sharing contract if it can keep the vendor’s share at a lower level.*

Proof: When $W_t$ or $\mu$ increases, $\partial(.)/\partial W_t$ and $\partial(.)/\partial \mu$ are always positive, except when $s$ is less than a certain level. $\partial U_c^{(\cdot)}/\partial(.)$ is always negative. ■

(ii) **Influence from client side**

*The client’s internal transaction price does not impact the client’s outsourcing contract selection, but the client’s liquidation cost does. If it is expensive to terminate the client’s*
previous in-house operations, the client has to ask a lower $P_{FP}, \alpha_{CP},$ or $P_{TG}$ in order to protect its utility.

Proof: $\frac{\partial(.)}{\partial P_C} = 0$ and $\frac{\partial(.)}{\partial K_C} < 0.$

(iii) Influence from interaction between vendor and client

When the vendor has a larger cost advantage or higher negotiation power, the client has to suffer a higher level $P_{FP}, \alpha_{CP},$ or $P_{TG}.$

Proof: $\frac{\partial(.)}{\partial \lambda} < 0$ and $\frac{\partial(.)}{\partial \gamma} \geq 0.$

(iv) Influence from outside

When the capital cost increases, the cost-plus contract is the client’s appropriate choice. The competition among vendors will not impact $P_{FP}$ and $\alpha_{CP},$ but will have an effect on $P_{TG}.$

Proof: $\frac{\partial \alpha^*_{CP}}{\partial r} < 0.$ $\frac{\partial P^*_F}{\partial \alpha} = \frac{\partial \alpha^*_{CP}}{\partial \alpha} = 0,$ but $\frac{\partial P^*_T}{\partial \alpha} < 0.$

5. OUTSOURCING TIMING

The above section focuses on the client’s outsourcing contract selection and negotiation at moments $t_0^-$ and $t_0.$ Based on the results of contract selection and negotiation, we now consider how the client makes the decision of outsourcing timing at the moment $t_0^+, i.e.,$ the client should outsource the previous in-house activities immediately or hold them for sometime. To introduce the influence of timing under cost uncertainty, we extend the aforementioned basic setting to a general framework by assuming operating cost $W_t$ evolves over time as a general Brownian motion (GBM), which is the continuous-time formulation of the random walk: $dW_t = \mu W_t dt + \sigma W_t dB_t,$ where $dB_t$ denotes a standard GBM process; $\mu$ is still the shift rate of expected future change;
\( \sigma \) describes the uncertainty rate of such a process. When \( \sigma = 0 \), \( W_t \) converges to the setting discussed before this section. The current GBM setting is the standard setting in real options theory and also a good first approximation for uncertainties (Kamien & Li, 1990; Abel & Eberly, 1994; Dixit & Pindyck, 1994).

Since the client has the freedom to decide the outsourcing time, such a managerial flexibility can be treated as an option from a client’s standpoint, i.e., outsource now or later. According to real options theory, we can calculate the client’s value with the option to outsource as follows:

Under the fixed-price contract, for a given fixed price \( P_{FP} \) (note that it is not just the optimal negotiated \( P_{FP}^* \)), the client’s value is:

\[
V_c(W_C) = \begin{cases} 
  a_{FP}^1 (W_C)^{\beta_1} + \frac{P_C}{r} - \frac{W_C}{r - \mu} & \text{if } W_C < W_{FP}^* \\
  \frac{P_C - P_{FP}}{r} - K_C & \text{if } W_C \geq W_{FP}^* 
\end{cases}
\]

(13)

where \( W_{FP}^* = \frac{\beta_1}{\beta_1 - 1} \left( \frac{P_{FP}}{r} + K_C \right)(r - \mu) \).

(14)

\( a_{FP}^1 \) and \( \beta_1 \) can be derived from the standard real options approach (See Appendix C). The top part of Eq. (13) can be interpreted as: when the client’s in-house operating cost is lower than the outsourcing threshold \( W_{FP}^* \), the client will keep the operation in-house by holding the traditional net present value \( \frac{P_C}{r} - \frac{W_C}{r - \mu} \) and the value of option to outsource \( a_{FP}^1 (W_C)^{\beta_1} \). The bottom part of Eq. (13) can be interpreted as: when the client’s in-house operating cost is too high (higher than the threshold \( W_{FP}^* \)), the client will terminate the previous in-house operation by paying the liquidation cost \( K_C \) and obtain the benefit of outsourcing \( \frac{P_C - P_{FP}}{r} \) by signing the fixed-price contract.
Under the cost-plus outsourcing contract, for a given profit margin $\alpha_{cp}$ (note that it is not just the optimal negotiated $\alpha_{cp}^*$), the client’s value is:

$$V_c(W_c) = \begin{cases} a_{cp}^1(W_c)^{\beta_i} + \frac{P_c}{r} - \frac{W_c}{(r - \mu)} & \text{if } W_c < W_{cp}^* \\
\frac{P_c}{r} - (1 + \alpha_{cp}) \frac{\lambda W_c}{(r - \mu)} - K_c & \text{if } W_c \geq W_{cp}^* \end{cases}$$

(15)

where

$$W_{cp}^* = \frac{\beta_i}{\beta_i - 1} K_c \frac{r - \mu}{1 - (1 + \alpha_{cp}) \lambda}.$$  

(16)

$a_{cp}^1$ and $\beta_i$ can be derived from the standard real options approach (see Appendix C). Note that if $\alpha_{cp} > 1/\lambda - 1$, $W_{cp}^* < 0$, i.e., the vendor is too dominant (cost can never be negative). As a result, the client will never outsource if the vendor is extremely dominant in the outsourcing market.

Under the gain-sharing outsourcing contract, for a given target price $P_{TG}$ (note that it is not just the optimal negotiated $P_{TG}^*$), the client’s value is:

$$V_c(W_c) = \begin{cases} a_{gs}^1(W_c)^{\beta_i} + \frac{P_c}{r} - \frac{W_c}{(r - \mu)} & \text{if } W_c < W_{gs}^* \\
\frac{P_c - P_{TG}^*}{r} - (1 - s)(1 + \alpha) \frac{\lambda W_c}{(r - \mu)} - K_c & \text{if } W_c \geq W_{gs}^* \end{cases}$$

(17)

where

$$W_{gs}^* = \frac{\beta_i}{\beta_i - 1} \frac{(P_{TG}^* + K_c)(r - \mu)}{r(1 - (1 - s)(1 + \alpha) \lambda)}.$$  

(18)

$a_{gs}^1$ and $\beta_i$ can be derived from the standard real options approach (see Appendix C). Note that if $\alpha > 1/(\lambda(1 - s)) - 1$, $W_{gs}^* < 0$, i.e., the outsourcing market’s profit margin is too high to be acceptable by the client (cost can never be negative). As a result, the client will never outsource if the competition among vendors is extremely weak.

In short, when the client’s in-house operating cost is low enough ($W_c < W_i^*$, $i \in \{FP, CP, GS\}$), the top parts of Eqs. (13, 15, 17) are the net present value of operating in-house
with the option to outsource in hand. However, when the client’s in-house operating cost \( W_c \) is higher than the threshold \( W_c^* \), the client would exercise the option to outsource.

If the client can obtain the optimal negotiated fixed price \( P_{FP}^* \) under the fixed-price contract, the optimal negotiated profit margin \( \alpha_{FP}^* \) under the cost-plus contract, or the optimal negotiated target price \( P_{GS}^* \) under the gain-sharing contract, then the client’s values under different outsourcing contracts converge to:

\[
V_c(W_c) = \begin{cases} 
 a_{NG}^l(W_c)^{\beta_1} + \frac{P_c}{r} - \frac{W_c}{r - \mu} & \text{if } W_c < W_{NG}^* \\
 \frac{P_c}{r} \left(1 + \gamma \left(\frac{1}{\lambda} - 1\right)\right) - \frac{W_c}{r - \mu} - (1 - \gamma)K_c & \text{if } W_c \geq W_{NG}^* 
\end{cases}
\]

\[
W_{NG}^* = \frac{\beta_1}{\beta_1 - 1} K_c \frac{r - \mu}{1 - \lambda}.
\]

\( a_{NG}^l \) and \( \beta_1 \) can be derived from the standard real options approach (see Appendix C). Again, it is interesting to see that the format of contract (fixed-price, cost-plus, or gain-sharing) has no impact on the client’s timing to outsource, as long as the client can achieve the optimal negotiated results under different outsourcing contracts.

Considering Eqs. (14, 16, and 18), we get Proposition 3 as follows (see Appendix D):

**Proposition 3:**

\[
\frac{\partial W_{FP}^*}{\partial \sigma} > 0, \quad \frac{\partial W_{CP}^*}{\partial \sigma} \begin{cases} > 0, \text{ if } \alpha_{CP} < \frac{1}{\lambda} - 1 \\
\leq 0, \text{ if } \alpha_{CP} \geq \frac{1}{\lambda} - 1 \end{cases}, \quad \frac{\partial W_{GS}^*}{\partial \sigma} \begin{cases} > 0, \text{ if } \alpha < \frac{1}{\lambda(1-s)} - 1 \\
\leq 0, \text{ if } \alpha \geq \frac{1}{\lambda(1-s)} - 1 \end{cases}.
\]

\[
\frac{\partial W_{FP}^*}{\partial K_c} > 0, \quad \frac{\partial W_{CP}^*}{\partial K_c} \begin{cases} > 0, \text{ if } \alpha_{CP} < \frac{1}{\lambda} - 1 \\
\leq 0, \text{ if } \alpha_{CP} \geq \frac{1}{\lambda} - 1 \end{cases}, \quad \frac{\partial W_{GS}^*}{\partial K_c} \begin{cases} > 0, \text{ if } \alpha < \frac{1}{\lambda(1-s)} - 1 \\
\leq 0, \text{ if } \alpha \geq \frac{1}{\lambda(1-s)} - 1 \end{cases}.
\]
\[
\frac{\partial W^*_i}{\partial \lambda} = 0, \quad \frac{\partial W^*_{CP}}{\partial \lambda} > 0, \quad \text{and} \quad \frac{\partial W^*_{GS}}{\partial \lambda} > 0.
\]

Note that the exogenous \( \alpha \) can be interpreted as the degree of competition among vendors in the outsourcing market. Conventional wisdom says that when uncertainty is high, it is better to defer the decision making. However, Proposition 3 reveals that the client sometimes can outsource earlier when facing high uncertainty.

Based on Proposition 3, we obtain Theorem 4:

**Theorem 4 (outsourcing timing):**

(i) When the future cost change is fuzzy (\( \sigma \) becomes larger), a fixed price always encourages the client to outsource early; a cost-plus or gain-sharing contract encourages the client to outsource early as long as vendor competition is fierce (\( \alpha_{CP} < 1/\lambda - 1 \) or \( \alpha < 1/[\lambda(1-s)]-1 \)).

Proof: When \( \sigma \) increases, the outsourcing threshold \( W^*_i \) (\( i \in \{FP, CP, GS\} \)) decreases, for \( \alpha \) in appropriate range, respectively. The likelihood of \( W^*_C \geq W^*_i \) increases. However, when \( \alpha_{CP} \geq 1/\lambda - 1 \), or \( \alpha \geq 1/[\lambda(1-s)]-1 \), \( W^*_{CP} \leq 0, W^*_{GS} \leq 0 \), the client will not outsource under a cost-plus or gain-sharing contract. ■

(ii) When the client’s previous in-house operating liquidation cost is high (\( K_C \) becomes large), the client always hesitates to outsource under a fixed-price contract; however, as long as vendor competition is fierce (\( \alpha_{CP} < 1/\lambda - 1 \) or \( \alpha < 1/[\lambda(1-s)]-1 \)), the client prefers to outsource earlier under a cost-plus or gain-sharing contract.

The proof is similar to the above case.
(iii) The larger the vendor’s cost advantage is, the earlier the client will outsource the in-house operations under a cost-plus or gain-sharing contract. The client’s outsourcing timing is robust to the vendor’s cost advantage under a fixed-price contract.

Proof: When the vendor’s cost advantage over the client becomes huge (i.e., $\lambda$ becomes small), $W_{CP}^*$ and $W_{GS}^*$ decrease and the likelihood of $W_C \geq W_i^*$ ($i \in \{CP, GS, NG\}$) increases. As a result, the client will outsource earlier. ■

6. THE PRACTICAL GUIDANCE
With all the aforementioned analyses and theorems, we now discuss how a client could apply these principles in an outsourcing process.

The general principle
No outsourcing contract is necessarily superior to others under effective negotiation. When the client and the vendor are risk neutral, their negotiation will reach the same outcome regardless of the type of contract. Considering Eqs. (12 and 20), as long as the client can obtain the optimal negotiated $P_{FP}^*$, $\alpha_{CP}^*$, or $P_{TG}^*$, the three common outsourcing contracts (fixed-price, cost-plus, and gain-sharing) provide the same expected utilities and option values to the client. Based on the known information (e.g., $P_C$, $r$, and $W_{h_i}$) and practical observations (e.g., $\gamma$, $\alpha$, $\mu$, and $\sigma$), the client can estimate these optimal negotiated results.

In reality, however, the client may not have all the required information (e.g., $P_C$, $r$, $W_{h_i}$, $\gamma$, $\alpha$, $\mu$, and $\sigma$) to negotiate $P_{FP}^*$, $\alpha_{CP}^*$, or $P_{TG}^*$. Theorem 2 sets the tone for the client’s outsourcing contract selection: the client must encourage the vendor’s cost reduction endeavors. The following principles at $t_0^-$, $t_0$, and $t_0^+$ may help the client’s decision making step-by-step.

$t_0^-$: **choose an outsourcing contract**
According to Theorem 1, the fixed-price contract should be the client’s choice under the following situations:

1. The outsourcing market’s profit margin is unclear;
2. The changes in future cost are uncertain.

While Theorem 2 points out that the cost-plus contract does not encourage the vendor to reduce its operating costs, Theorem 1 implies that the cost-plus contract should be the client’s choice under the following situations:

1. The capital cost is high;
2. The outsourcing market’s profit margin is unclear.

According to Theorem 1, the gain-sharing contract should be the client’s choice under the following situations:

1. The vendor’s operating cost is expected to increase;
2. The competition among vendors is weak.

$t_0$: negotiate the outsourcing contract

To negotiate the outsourcing contract, the client should follow Theorem 3.

1. When the cost to terminate the client’s in-house operation is expensive, the client should require a lower level $P_{FP}, \alpha_{CP},$ or $P_{TG}$.
2. When the vendor’s cost advantage over the client’s is significant, the client would have to take a higher level $P_{FP}, \alpha_{CP},$ or $P_{TG}$.
3. For the fixed-price contract, if the cost is expected to increase, the client has to suffer a high $P_{FP}$; if the cost is expected to decrease, the client may require a low $P_{FP}$. 
(4) For the cost-plus contract, a) if the cost is expected to increase (decrease), the client is more likely to negotiate a low (high) \( P_{cp} \); b) if the competition among vendors is strong (weak), the client is more likely to get a low (high) \( P_{cp} \).

(5) For the gain-sharing contract, a) if the cost is expected to increase (decrease), the client is more likely to get a low (high) \( P_{gs} \); b) if the competition among vendors is strong (weak), the client may ask a high (low) \( P_{gs} \); c) the client should provide the vendor a large enough share ratio in negotiation to encourage the vendor to reduce operating costs.

\( t_0^* \): **outsource now or later**

Now the client faces the decision: outsourcing now or later. According to Theorem 4, the outsourcing timing is relevant to the selected outsourcing contract:

For the fixed-price contract:

(1) The client should outsource early if the future cost change is uncertain;

(2) The client should hold the option to outsource when the liquidation cost is high;

(3) The client’s outsourcing timing is robust to the vendor’s cost advantage.

For the cost-plus or gain-sharing contract:

(1) The client should outsource early if the vendor’s cost advantage is significant;

(2) As long as the competition among vendors is weak, the client should hold the option to outsource when a) the future cost change is uncertain; b) the liquidation cost is high.

**7. MANAGERIAL IMPLICATIONS AND CONCLUSIONS**

While the contract type choice is a popular topic in the literature of outsourcing, existing research on outsourcing contract selection is predominantly descriptive and the output is too vague to be applied in practice. For example, Barthelemy (2003) designs a roadmap of how the client-vendor
relationship will change over its life cycle: fixed-price contract may be used at the beginning of outsourcing; then the pricing could switch to cost-plus as the relationship develops; eventually, the contract could call for a change to a gain-sharing arrangement so that the client and the vendor have a joint stake in the outcome. Such an argument may mislead readers or managers to believe that some outsourcing contracts should dominate others. Unfortunately, the practice in reality enhances this misunderstanding. Table 1 summarizes French and USA governmental outsourcing contracts in defense industries. It is obvious that fixed-price contracts dominate other outsourcing contracts (especially in France) and gain-sharing contracts always play a weak role.

Table 1. Comparison of governmental defense outsourcing contracts in France and USA

<table>
<thead>
<tr>
<th>France</th>
<th>USA</th>
</tr>
</thead>
<tbody>
<tr>
<td>DGA Outsourcing Contracts</td>
<td>Pentagon Outsourcing Contracts</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>18 defense companies</th>
<th>Lockheed Martin</th>
<th>Boeing</th>
<th>Raytheon Co.</th>
<th>Northrop Grumman</th>
<th>General Dynamics</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fixed-price</td>
<td>97.92%</td>
<td>46.77%</td>
<td>70.25%</td>
<td>57.94%</td>
<td>49.55%</td>
<td>60.02%</td>
</tr>
<tr>
<td>Cost-plus</td>
<td>2.08%</td>
<td>49.68%</td>
<td>27.42%</td>
<td>37.35%</td>
<td>42.48%</td>
<td>38.87%</td>
</tr>
<tr>
<td>Gain-sharing</td>
<td>0%</td>
<td>3.55%</td>
<td>2.33%</td>
<td>4.71%</td>
<td>7.97%</td>
<td>1.11%</td>
</tr>
</tbody>
</table>

The source of American data is from http://www.publicintegrity.org/pns/ The source of French data is from Délégation Générale pour l’Armement (DGA) http://www.defense.gouv.fr/dga

To reveal the relationship among the three popular outsourcing contracts, fixed-price, cost-plus and gain-sharing, in this research we develop an analytical model with a real options approach to evaluate them through contract price, profitability, capital cost, liquidation cost, negotiating power, and the uncertainty of operating cost. This work constitutes the first attempt to thoroughly compare the three outsourcing contracts subject to irreversible outsourcing investment.
and uncertain future operating cost by analytical methods rather than qualitative studies (which may lack rigorous verification and validation) and empirical observations (which may be lack of generalization). Our major results are summarized as follows:

First, if the client can negotiate effectively, the three outsourcing contracts converge to the same utility and the same option value. In the extant outsourcing literature, there are conflicting arguments on outsourcing contract selection. Our results reveal that no contract is necessarily superior to others if the client has enough information.

This finding can explain the differences in the choice of contract types in USA and France. In the US, authorities rely on both law (the Truth In Negotiation Act) and on large and efficient audit controls (through the General Accountability Office) to monitor governmental contracts. This institutional environment favors the public authorities’ accurate knowledge on vendor realization costs. In France, no law demands a revelation of information for fear of being indicted and, above all, the efficiency of audit controls are relative, although efforts are currently run in that perspective (Beaufils et al., 2004). The DGA’s almost systematic choice for the fixed-price contract can be explained by the wish to provide incentives to perform in a context of both information asymmetry on the realization costs and weaknesses of the audit controls (Oudot, 2006). The reason why gain-sharing contracts are rare in governmental outsourcing can be explained as: governmental clients cannot acquire all necessary information to estimate whether outsourcing projects will be cost-overrunning or cost-saving. It is difficult to get extra budgets from government bureaucracy to cover an unexpected cost-overrun. As a result, governmental outsourcing contracts prefer fixed-price to gain-sharing.

Second, given the asymmetric information problem, the client may be unable to acquire the optimal negotiation results, so that the three contracts do generate different values to the client. By analytically breaking the client’s decision making process into pieces ($t^0_0$, $t_0$, and $t^+_0$), we explore practical insights into client’s contract selection, contract negotiation, and outsourcing timing.
valuation given limited information. Outsourcing timing is often a neglected factor in the outsourcing decision making literature. Our research fills this gap and may help a pre-outsourcing firm choose the right outsourcing contract and its exercise timing.

The major limitation of the research is that it only focuses on the client’s pursuit of cost reduction. Actually, while cost-saving is one of the major criteria of a client’s outsourcing decision, there are several other factors, such as access to state-of-the-art technology, focus on the core competence, or improve quality and productivity. Future research can incorporate these features into the model and extend this research to an outsourcing decision in which non-costs dimensions are incorporated.

APPENDIX A: Proof of Eq. (8)

We rewrite Eqs. (2) and (3) here:

\[ U_y = E[\int_0^\infty (P_{FP} - W_t) e^{-r(t-\tau)} d\tau] = \frac{W_t}{r} - \frac{P_{FP}}{r - \mu} \]

\[ U_C^{FP}(t) = E[\int_t^\infty (P_C - P_{FP}) e^{-r(t-\tau)} d\tau] - K_C = \frac{P_C - P_{FP}}{r} - K_C. \]

From Eq.(1),

\[ B(P_{FP}) = \left(\frac{P_{FP}}{r} - \frac{W_t}{r - \mu}\right)^{\gamma} \left(\frac{P_C - P_{FP}}{r} - K_C - \left(\frac{P_C}{r} - \frac{W_t}{(r - \mu)\lambda}\right)^{1-\gamma} \right) \]

\[ = \left(\frac{P_{FP}}{r} - \frac{W_t}{r - \mu}\right)^{\gamma} \left(\frac{W_t}{(r - \mu)\lambda} - \left(\frac{P_{FP}}{r} + K_C\right)^{1-\gamma} \right). \]

Find the first order condition of \(\log B\) w.r.t. \(P_{FP}\),

\[ \log B(P_{FP}) = \gamma \log \left(\frac{P_{FP}}{r} - \frac{W_t}{r - \mu}\right) + (1 - \gamma) \log \left(\frac{W_t}{(r - \mu)\lambda} - \left(\frac{P_{FP}}{r} + K_C\right)\right) \]

\[ \frac{\partial \log B(P_{FP})}{\partial P_{FP}} = \frac{\gamma}{r\left(\frac{P_{FP}}{r} - \frac{W_t}{r - \mu}\right)} + \frac{-(1 - \gamma)}{r\left(\frac{W_t}{(r - \mu)\lambda} - \left(\frac{P_{FP}}{r} + K_C\right)\right)} = 0, \]
which results \( P^*_{FP} = (1 + \gamma(\frac{1}{\lambda} - 1)) \frac{r}{r - \mu} W_t - \gamma K_c r \).

**Proof of Eq. (9)**

\[
U^c_V(t) = E \int_t^\infty (P_{CP} - W) e^{-r(t-\tau)} d\tau = \alpha_{CP} \frac{W_t}{r - \mu},
\]

\[
U^c_\tau(t) = E \int_t^\infty (P - P_{CP}) e^{-r(t-\tau)} d\tau - K_c = \frac{P_c}{r} - K_c - (1 + \alpha_{CP}) \frac{W_t}{r - \mu},
\]

\[
B(\alpha_{CP}) = (\alpha_{CP} \frac{W_t}{r - \mu})^\gamma (\frac{P_c}{r} - K_c - (1 + \alpha_{CP}) \frac{W_t}{r - \mu} - (\frac{P_c}{r} - \frac{W_t}{(r - \mu)\lambda}))^{1-\gamma}
\]

\[
= (\alpha_{CP} \frac{W_t}{r - \mu})^\gamma ((\frac{1}{\lambda} - (1 + \alpha_{CP})) \frac{W_t}{r - \mu} - K_c)^{1-\gamma}
\]

Similarly as the proof of Eq.(8), we have \( \alpha^*_{CP} = \gamma(-K_c \frac{r - \mu}{W_b} + \frac{1}{\lambda} - 1) \)

**Proof of Eq. (10)**

\[
U_V = E \int_t^\infty (P^c_{GS} - W) e^{-r(t-\tau)} d\tau = E \int_t^\infty (P_{TG}^s + (1 - s)(1 + \alpha)W - W) e^{-r(t-\tau)} d\tau
\]

\[
= \frac{P_{TG}^s}{r} + [(1 - s)(1 + \alpha) - 1] \frac{W_t}{r - \mu}
\]

\[
U_\tau = E \int_t^\infty (P_c - P^c_{GS}) e^{-r(t-\tau)} d\tau - K_c = E \int_t^\infty (P_c - P_{TG}^s(1 - s)(1 + \alpha)W e^{-r(t-\tau)} d\tau - K_c
\]

\[
= \frac{P_c - P_{TG}^s}{r} - K_c - (1 - s)(1 + \alpha)W_t
\]

From Eq. (1),
\[ B(P_{TG}, s) = \left\{ \frac{P_{TG} s}{r} + [(1 - s)(1 + \alpha) - 1] \frac{W_t}{r - \mu} \right\}^\gamma \left\{ \frac{P_c - P_{TG} s}{r} - K_c - \frac{(1 - s)(1 + \alpha)W_t}{r - \mu} - \left( \frac{P_c - W_t}{r - \mu} \right) \right\}^{1 - \gamma} \]

\[ = \left\{ \frac{P_{TG} s}{r} + [(1 - s)(1 + \alpha) - 1] \frac{W_t}{r - \mu} \right\}^\gamma \left\{ \left( \frac{1}{\lambda} - (1 - s)(1 + \alpha) \right) \frac{W_t}{r - \mu} - \left( \frac{P_{TG} s}{r} + K_c \right) \right\}^{1 - \gamma} \]

Similarly as the proof of Eq. (8), we have \( P_{TG}^* = \frac{r}{s} \left\{ [(1 + \alpha)s - \alpha + \gamma(\frac{1}{\lambda} - 1)] \frac{W_t}{r - \mu} - K_c \right\}. \)

**APPENDIX B: Proof of Proposition 2**

The proof follows from Eqs. (8), (9a) and (10a).

\[ P_{GS}^* = (1 - s)(1 + \alpha) \frac{-\mu}{r - \mu} W_t + (1 + \gamma(\frac{1}{\lambda} - 1)) \frac{r}{r - \mu} W_t - \gamma K_c r \]

\[ = P_{FP}^* - (1 - s)(1 + \alpha) \frac{\mu}{r - \mu} W_t = P_{CP}^* + \frac{\mu}{r - \mu} \left\{ [\alpha(s - 1) + s + \gamma(\frac{1}{\lambda} - 1)] W_t - \gamma K_c (r - \mu) \right\} \]

\[ P_{GS}^* - P_{FP}^* = -(1 - s)(1 + \alpha) \frac{\mu}{r - \mu} W_t \]

\[ P_{GS}^* - P_{CP}^* = \frac{\mu}{r - \mu} \left\{ [\alpha(s - 1) + s + \gamma(\frac{1}{\lambda} - 1)] W_t - \gamma K_c (r - \mu) \right\} \]

\[ P_{FP}^* - P_{CP}^* = \frac{\mu}{r - \mu} \left\{ [1 + \gamma(\frac{1}{\lambda} - 1)] W_t - \gamma K_c (r - \mu) \right\} \]

So Proposition 2 follows from the above results.

**APPENDIX C: Proof of Eqs (13) and (14)**

Before outsourcing occurs, the client’s previous in-house operations produce a net cash flow \((P_c - W_c)dt\). We represent the value of option to outsource as \(V(W_c)\). Hence as shown in Pindyck and Dixit (Chapter 4), in the continuation region the Bellman equation is:

\[ [rV(W_c) - (P_c - W_c)]dt = E(dV) \quad (C.1) \]

Using Ito’s Lemma, the Bellman equation becomes:
\[
\frac{1}{2} \sigma^2 W_c^2 V'(W_c) + \mu W_c V'(W_c) - rV + P_c - W_c = 0
\]  
(C.2)

In addition, \(V(W_c)\) should satisfy the following boundary conditions:

\[
V(0) = \frac{P_c}{r}
\]  
(C.3)

\[
V(W_{fp}^*) = \frac{P_c - P_{fp}}{r} - K_c
\]  
(C.4)

\[
\frac{\partial V(W_{fp}^*)}{\partial W_c} = \frac{\partial \left( \frac{P_c - P_{fp}}{r} - K_c \right)}{\partial W_c} = 0
\]  
(C.5)

Condition (C.3) arises since the client has internal transaction profits when \(W_c = 0\). Conditions (C.4) and (C.5) are smooth pasting and value matching conditions coming from optimality. The general solution for equation (C.2) must take the form

\[
V(W_c) = a^1 W_c^{\beta_1} + a^2 W_c^{\beta_2},
\]

where \(a^1, a^2\) are constants to be determined, and \(\beta_1 = \frac{1}{2} - \frac{\mu}{\sigma^2} + \sqrt{(\frac{\mu}{\sigma^2} - \frac{1}{2})^2 + \frac{2r}{\sigma^2}} > 1\), \(\beta_2 = \frac{1}{2} - \frac{\mu}{\sigma^2} - \sqrt{(\frac{\mu}{\sigma^2} - \frac{1}{2})^2 + \frac{2r}{\sigma^2}} < 0\). To satisfy the condition (C.3), we must have \(a^2 = 0\), so the general solution must have the form \(V(W_c) = a^1 W_c^{\beta_1}\).

\[
V_c(W_c) = \begin{cases} 
  a_{fp}^1 (W_c)^{\beta_1} + \frac{P_c}{r} - \frac{W_c}{(r - \mu)} & \text{if } W_c < W_{fp}^* \\
  \frac{P_c - P_{fp}}{r} - K_c & \text{if } W_c \geq W_{fp}^*
\end{cases}
\]

where \(W_{fp}^* = \frac{\beta_1}{\beta_1 - 1} (\frac{P_{fp}}{r} + K_c)(r - \mu)\) and \(a_{fp}^1 = \frac{\beta_1}{r - \mu} \frac{(W_{fp}^*)^{1 - \beta_1}}{r - \mu} \).

Eqs. (15) to (20), \(a_{(i)}^{(1)}, \beta_i\) and \(W_{(i)}^*\) can be derived from the similar approach.
APPENDIX D: Proof of Proposition 3

To prove it, we need the following result:

\[
\frac{\partial \beta_1}{\partial \sigma} = \left(2 \mu + \frac{(\mu^2 - \frac{1}{2}) - 2 \mu + \frac{2r}{\sigma^3}}{\sqrt{(\mu^2 - \frac{1}{2})^2 + 2r \sigma^2}}\right) = \frac{2\mu}{\sigma^3 \sqrt{(\mu^2 - \frac{1}{2})^2 + 2r \sigma^2}}\]  

If \( \mu = 0 \), then \( \frac{\partial \beta_1}{\partial \sigma} = \frac{-2r}{\sigma^3} < 0 \); if \( \mu < 0 \), then \( \frac{\mu}{\sigma^2} - \frac{1}{2} + \frac{r}{\mu} < 0 \), hence \( \frac{\partial \beta_1}{\partial \sigma} < 0 \); if \( \mu > 0 \), since \( \mu < r \), we have

\[
\frac{\mu}{\sigma^2} - \frac{1}{2} + \frac{r}{\mu} > 0 \text{ and } (\frac{\mu}{\sigma^2} - \frac{1}{2} + \frac{r}{\mu})^2 = \frac{(\mu^2 - \frac{1}{2})^2 + 2r \sigma^2}{\sigma^2} + \frac{r (r - 1)}{\mu^2} > (\frac{\mu}{\sigma^2} - \frac{1}{2})^2 + \frac{2r}{\sigma^2}.
\]

Hence \( \frac{\partial \beta_1}{\partial \sigma} < 0 \).

\[
\frac{\partial W_{FP}^*}{\partial \sigma} = \frac{\partial W_{FP}}{\partial \beta_1} \frac{\partial \beta_1}{\partial \sigma} = \frac{-1}{(\beta - 1)^2} \left( \frac{P_{FP}}{r} + K_c \right) (r - \mu) \frac{\partial \beta_1}{\partial \sigma} > 0;
\]

\[
\frac{\partial W_{CP}^*}{\partial \sigma} = \frac{\partial W_{CP}}{\partial \beta_1} \frac{\partial \beta_1}{\partial \sigma} = \frac{-1}{(\beta - 1)^2} K_c \frac{1}{\lambda - (1 + \alpha_{CP})} \frac{\partial \beta_1}{\partial \sigma} \begin{cases} > 0, & \text{if } \alpha_{CP} < \frac{1}{\lambda} - 1 \\ \leq 0, & \text{if } \alpha_{CP} \geq \frac{1}{\lambda} - 1 \end{cases}
\]

\[
\frac{\partial W_{GS}^*}{\partial \sigma} = \frac{\partial W_{GS}}{\partial \beta_1} \frac{\partial \beta_1}{\partial \sigma} = \frac{-1}{(\beta - 1)^2} \left( \frac{P_{GS}}{r} + K_c \right) (r - \mu) \frac{\partial \beta_1}{\partial \sigma} \begin{cases} > 0, & \text{if } \alpha < \frac{1}{\lambda - s} - 1 \\ \leq 0, & \text{if } \alpha \geq \frac{1}{\lambda - s} - 1 \end{cases}
\]

From Eqs. (15,17,19), we have:

\[
\frac{\partial W_{FP}^*}{\partial K_c} = \frac{\beta_1}{\beta_1 - 1} (r - \mu) > 0
\]
\[
\frac{\partial W^*_C}{\partial K_c} = \frac{\beta_1}{\beta_1 - 1} \frac{r - \mu}{(1 + \alpha_{CP})\lambda} \begin{cases} 
< 0, \text{if } \alpha_{CP} < \frac{1}{\lambda} - 1 \\
\geq 0, \text{if } \alpha_{CP} \geq \frac{1}{\lambda} - 1 
\end{cases}
\]

\[
\frac{\partial W^*_G}{\partial K_c} = \frac{\beta_1}{\beta_1 - 1} \frac{r - \mu}{1 - (1 - s)(1 + \alpha)\lambda} \begin{cases} 
< 0, \text{if } \alpha < \frac{1}{\lambda(1 - s)} - 1 \\
\geq 0, \text{if } \alpha \geq \frac{1}{\lambda(1 - s)} - 1 
\end{cases}
\]

\[
\frac{\partial W^*_P}{\partial \lambda} = 0;
\]

\[
\frac{\partial W^*_C}{\partial \lambda} = \frac{\beta_1}{\beta_1 - 1} K_c \frac{(r - \mu)(1 + \alpha_{CP})}{(1 + (\alpha_{CP})\lambda)^2} > 0;
\]

\[
\frac{\partial W^*_G}{\partial \lambda} = \frac{\beta_1}{\beta_1 - 1} r \frac{(P_{TG} s + K_c r)(r - \mu)}{r(1 - s)(1 + \alpha)\lambda^2} (1 - s)(1 + \alpha) > 0.
\]

REFERENCES


